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DIFFRACTIVE DISSOCIATION IN THE DUAL MODEL

J.P. ADER and L. CLAVELLI *
Laboratoire de Physique Théorique **, *Bordeaux, France*

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We study the one twisted loop contribution to the five-point function in the Neveu-Schwarz model. The formulation obtained incorporates the requirements of duality and analyticity (Steinmann relations), provides a low-mass vector-pseudoscalar system predominantly as an S-wave $J^P = 1^+$ state near threshold, conserves approximately t-channel helicity and thus is an interesting candidate for the phenomenology of diffractive dissociation.

1. Introduction

The diffractive dissociation of π into $\rho\pi$ together with the question of a possible A_1 resonance in the $\rho\pi$ system has been the subject of a long, often heated controversy that shows little sign of abating.

On the theoretical side the quark model seems to predict unambiguously the existence of a resonant $^3P_1$ state with the quantum numbers of the A_1 ($J^{PC} = 1^{2+}$, $I = 1$). Good candidates for the $I = 0$ and $I = \frac{1}{2}$ SU(3) partners of the A_1 exist. In addition there are chiral symmetry arguments for the A_1 mass $m_A = \sqrt{2}\, m_\rho$.

Experimentally there was early observation of a broad peak in the $\rho\pi$ system centered near 1.1 GeV. However, a resonance interpretation of this peak was disfavored because the shape of the peak depended on the momentum transfer and because of the detailed behavior of the partial-wave phases. Very recently the resonance character of the A_1 has been supported by new data on 3-pion production and its backward production [1]. Whatever may be, the exact role of the background given by a non-resonant amplitude in this region remains an open question.

Theoretical work *** on this non-resonant peak is based on the Drell-Deck model

* Present address: Center for Theoretical Physics and Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742, USA.
** Equipe de Recherche associée au CNRS.
Postal address: Laboratoire de Physique Théorique, Université de Bordeaux I, Chemin du Solarium, 33170 Gradignan, France.
*** For a recent review see ref. [2].

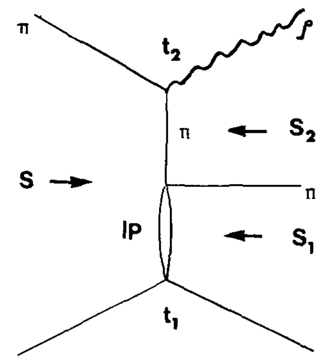

Fig. 1. Deck diagram for the diffractive dissociation of a pion into a system composed of a vector meson ρ and a pion.

[3], which can be described by the graph of fig. 1. The incident pion virtually dissociates into a $\rho\pi$ system and the virtual pion returns to its mass shell by diffractively scattering off the target particle. Because of the sharp diffractive peak the amplitude is dominantly contained in the region near $t_1 = 0$. Deck argued that the pion pole in t_2 would enhance the amplitude at small values of t_2 also. The combination of these two effects would kinematically limit the amplitude to small values of the $\rho\pi$ mass. The distribution as a function of the $\rho\pi$ mass grows from threshold as phase space opens up and then falls as a result of these kinematical effects producing a peak near threshold. The pion pole graph would also yield a t-channel helicity conservation near threshold as is experimentally observed [4].

However, the Deck model has never succeeded in reproducing the mass peak in a convincing way. The pion pole does not give a sharp enough cut-off in t_2 and more recent treatments introduce an exponential damping in t_2 justified on the basis of a reggeization of the pion. It was also noted [5] that near $t_1 = 0$ the pion pole is approximately cancelled due to the factor $s_1^{\alpha_{\mathrm{P}}(t_1)}$ and to the kinematic relation (as usual we define α_i as the trajectory of particle i and μ_π is the pion mass)

$$s_1 \simeq (\mu_\pi^2 - t_2)\frac{s}{s_2 - \mu_\pi^2} ,$$

which is valid near $t_1 = 0$. This being the case there seems to be no reason to neglect the graphs where the pomeron couples to the incident pion or to the ρ meson. These graphs do not clearly predict t-channel helicity conservation, however. In addition, duality has taught us to distrust single-particle exchange models and interference models where one simply adds graphs.

A step towards a correct dual treatment has been made recently by the Saclay group [6] who describe the $\pi \to \rho\pi$ + pomeron sub-amplitude by a sum of three beta functions containing the expected poles in (s_2, t_2), (t_2, u_2), and (u_2, s_2),

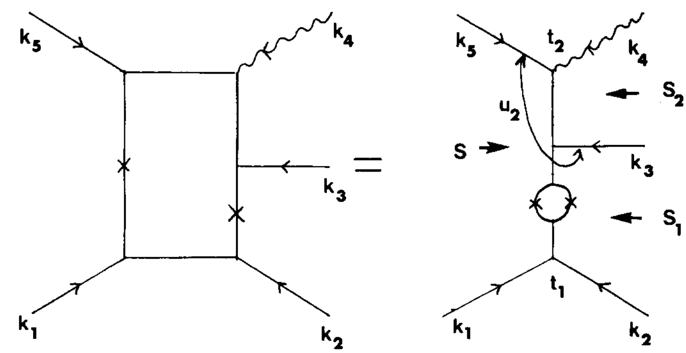

Fig. 2. Momentum diagram for the contribution of the single-loop graph to the five-point function and the equivalent 2-to-3 amplitude with channel invariants.

respectively. This approach, based on the Veneziano Born term, treats the diffractive dissociation as a planar amplitude. As a refinement of their attempt at dualization, these authors suggest for the future the use of the Virasoro-Shapiro amplitude [7] (VS) which has the correct structure of dual poles in (s_2, t_2, u_2) together. It is clear that even this is not fully appropriate since the VS amplitude corresponds to pomeron-pomeron elastic scattering in the dual model.

The correct lowest-order amplitude for diffractive dissociation is given by the twisted loop graph of fig. 2. In addition to having the correct topological structure of poles in s_2, t_2 and u_2 all dual to each other, the amplitude corresponding to fig. 2 will incorporate the f dominance and other good features of the dual pomeron. The dual amplitude we will write down will be a function of $\alpha_P(t_1)$, $\alpha_f(t_1)$, $\alpha_\pi(s_2)$ and $\alpha_\rho(u_2)$. It will be ghost free for $\alpha_P(0) = 2\alpha_\rho(0) = 2$. For physical values of the intercepts $\alpha_P(0) = 2\alpha_\rho(0) = 1$, the amplitudes will still be ghost free on the leading trajectories and will still have the correct analytic and topological structure. It is therefore this latter amplitude that we propose as a realistic model of the dissociation process.

Even after making this shift to physical intercepts the generalized Veneziano model is unsatisfactory for phenomenological purposes. This is because of the tachyons that exist in the model at $\alpha_P(t_1) = 0$ and $\alpha_\rho(u_2) = 0$. In order to avoid these poles in the physical region we will work with the Neveu-Schwarz model (NSM) [8] which has no tachyons on the leading pomeron and ρ, f trajectories. The NSM has poles at $\alpha_\pi(s_2) = 0, 2, 4, \ldots$. Thus with physical intercepts there is no resonance at the A_1 position $\alpha_\pi(s_2) = 1$. The model is therefore ideally suited to determine to what extent the dynamics of diffraction can build the broad bump near threshold in the $\pi\rho$ system.

In sect. 2 of this paper we calculate the twisted 5-point loop in the Neveu-Schwarz model with an external ρ meson. Sect. 3 is devoted to a study of the analytic structure in the single-Regge limit. This will provide a basis for the examination of the Deck phenomenon in the NSM. As we shall see the diffractive dissociation *via* the dual pomeron has a rich and complex structure which bears little comparison to the early Deck amplitudes. In sect. 4 we examine the double-Regge limit $s_1, s_2 \to \infty$, $s_1 s_2/s$ fixed.

A numerical evaluation and a phenomenological discussion of the amplitudes including a partial-wave analysis are presented in sect. 5. In particular, the existence of an A_1 effect is clearly shown. Some technical problems are discussed in appendices A and B.

2. Amplitude construction

We will calculate the graph of fig. 2, where particles 1, 2, 3 and 5 are π mesons represented in the NSM by

$$V_\pi(k_i, \rho_i) = :\mathrm{e}^{ik_i \cdot Q(\rho_i)} k_i \cdot H(\rho_i): \,, \tag{2.1}$$

where

$$H_\mu(X) = \sum_{n=0}^{\infty} (b_\mu^n X^{-n-1/2} + b_\mu^{n+} X^{n+1/2}) \,. \tag{2.2}$$

Particle 4 is the ρ meson represented by

$$V_\rho(k_4, \rho_4) = :\mathrm{e}^{ik_4 \cdot Q(\rho_4)} [k_4 \cdot H\epsilon \cdot H + \epsilon \cdot P]: \,, \tag{2.3}$$

with

$$P_\mu(X) = iX \frac{\mathrm{d}Q_\mu(X)}{\mathrm{d}X} = i \sum_{n=0}^{\infty} (n + \varepsilon) \left[\frac{\Gamma(n+2\varepsilon)}{n!}\right]^{1/2} (a_\mu^{n+} X^{n+\varepsilon} - a_\mu^n X^{-n-\varepsilon}) \,. \tag{2.4}$$

In (2.2) and (2.4) the b_μ and a_μ are operators obeying Fermi and Bose statistics, respectively; ϵ is the polarization vector of particle 4 and ε an infinitesimal quantity.

We are thus calculating the process $\pi\pi \to \pi\rho\pi$ *via* pomeron exchange. However, since in dual models, as in nature, the pomeron is a factorizing singularity our result will be proportional to a realistic dual amplitude for $\pi\mathrm{p} \to \pi\rho\mathrm{p}$. The constant of proportionality will be the pomeron form factor to the $\pi\pi$ state divided by the pomeron form factor to the $\mathrm{p}\bar{\mathrm{p}}$ state. This depends only on t_1, the momentum transfer carried by the pomeron, and is easily measured experimentally. The kinematics of fig. 1 is such that

$$k_1 \cdot k_2 = -\alpha_\rho(t_1) \,, \tag{2.5a}$$

$$k_1 \cdot k_3 = \alpha_\pi(s_1) + \alpha_\pi(t_1) - \alpha_\pi(t_2) \,, \tag{2.5b}$$

$$k_1 \cdot k_4 = \alpha_\pi(s) - \alpha_\pi(s_1) + \alpha_\pi(t_2)\,, \tag{2.5c}$$

$$k_1 \cdot k_5 = -\alpha_\rho(s)\,, \tag{2.5d}$$

$$k_2 \cdot k_3 = -\alpha_\rho(s_1)\,, \tag{2.5e}$$

$$k_2 \cdot k_4 = -\alpha_\pi(s) + \alpha_\pi(s_1) + \alpha_\pi(s_2)\,, \tag{2.5f}$$

$$k_2 \cdot k_5 = \alpha_\pi(s) - \alpha_\pi(s_2) + \alpha_\pi(t_1)\,, \tag{2.5g}$$

$$k_3 \cdot k_4 = -\alpha_\pi(s_2)\,, \tag{2.5h}$$

$$k_3 \cdot k_5 = \alpha_\pi(s_2) + \alpha_\pi(t_2) - \alpha_\rho(t_1)\,, \tag{2.5i}$$

$$k_4 \cdot k_5 = -\alpha_\pi(t_2)\,. \tag{2.5j}$$

For convenience the Regge slope has been normalized to $\frac{1}{2}$. Otherwise each momentum should be multiplied by $\sqrt{2\alpha'}$. These relations hold for the unphysical intercepts of the NSM. However, after having written the amplitude in terms of these trajectory functions we will for phenomenological purposes use physical intercepts.

The loop graph of fig. 2 is given by a trace over all the intermediate excited states of the model. Such traces with external excited states are most conveniently done by the method of ref. [9], with the loop momentum integration being given by the trace over the zeroth (translational) mode. For further details the reader can refer to that article.

$$A_5 = g^5 \int_0^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \frac{\mathrm{d}\rho_5}{\rho_5} \int_0^{\rho_5} \frac{\mathrm{d}\rho_4}{\rho_4} \int_0^{\rho_4} \frac{\mathrm{d}\rho_3}{\rho_3} \int_0^{\rho_3} \frac{\mathrm{d}\rho_2}{\rho_2}\, S(w)$$
$$\times\, \mathrm{Tr}\{w^{L_0} \Omega V_\pi(k_1, 1)\, V_\pi(k_2, \rho_2)\, \Omega V_\pi(k_3, \rho_3)\, V_\rho(k_4, \rho_4)\, V_\pi(k_5, \rho_5)\}\,. \tag{2.6}$$

Here L_0 is the dual Hamiltonian and Ω is the twist operator. We evaluate the trace in the critical dimensionality of space-time (10 for the NSM) and with the unphysical masses of the ρ and π, all invariants being absorbed into trajectory functions by eqs. (2.5a–j). In (2.6), $S(w)$ is a factor from projecting the circulating states onto the physical subspace:

$$S(w) = w^{1/2} \prod_{m=1}^{\infty} \left(\frac{1 - w^m}{1 + w^{m-1/2}}\right)^2 . \tag{2.7}$$

Details on trace calculation are given in appendix A. To study the pomeron singularity it is customary to make the Jacobi imaginary transformation

$$\rho_i \to \rho_i' \equiv \mathrm{e}^{i\theta_i} \equiv \exp \frac{2\pi i \ln \rho_i}{\ln w}\,, \tag{2.8a}$$

$$w \to w' \equiv r^2 \equiv \exp \frac{4\pi^2}{\ln w}\,. \tag{2.8b}$$

In the limit in which the pomeron cross-channel energy becomes asymptotic the loop integral is dominated by the region near $r = 0$. Therefore, we can expand the integrand to lowest order in r but keeping all orders in $r\alpha(s)$ and $r\alpha(s_1)$. It is also convenient to redefine the angular variables:

$$\theta_3 = \sigma = \gamma - \tfrac{1}{2}\theta_4 + \tfrac{1}{2}\theta_2 \,, \tag{2.9a}$$

$$\theta_4 = \theta_4' + \sigma \,, \tag{2.9b}$$

$$\theta_5 = \theta_5' + \sigma \,. \tag{2.9c}$$

Then, dropping all primes, the amplitude becomes

$$\begin{aligned} A_5 = g^5 \int_0^1 \mathrm{d}r\, r^{-\alpha_{\mathrm{P}}(t_1)} \int_0^{2\pi} \mathrm{d}\gamma \int_0^{2\pi} \mathrm{d}\theta_2 \int_0^{2\pi} \mathrm{d}\theta_5 \int_0^{\theta_5} \mathrm{d}\theta_4 (2\sin\tfrac{1}{2}\theta_2)^{-\alpha_{\mathrm{f}}(t_1)} \\ \times (2\sin\tfrac{1}{2}(\theta_5-\theta_4))^{-\alpha_\pi(t_2)} (2\sin\tfrac{1}{2}\theta_4)^{-\alpha_\pi(s_2)} (2\sin\tfrac{1}{2}\theta_5)^{-\alpha_\rho(u_2)} \\ \times \exp[-8r\alpha(s)\sin\tfrac{1}{2}\theta_2\sin\tfrac{1}{2}(\theta_5-\theta_4)\cos(\gamma+\tfrac{1}{2}\theta_5)] \\ \times \exp[-8r\alpha(s_1)\sin\tfrac{1}{2}\theta_2\sin\tfrac{1}{2}\theta_4\cos\gamma] \left[\left(\frac{r^2}{w}\right)^{D/48}\left(\frac{\ln w}{2\pi}\right)^3 T_b\right] . \end{aligned} \tag{2.10}$$

The trace T_b is given in appendix A (*cf.* eq. (A.23)). In the limit $r \to 0$ one finds

$$\chi^{(1/2)}(\rho_j/\rho_i, w) \simeq -\frac{\pi}{\ln w \sin\frac{1}{2}(\theta_j-\theta_i)} \,, \tag{2.11}$$

$$\chi_{\mathrm{T}}^{(1/2)}(\rho_j/\rho_i, w) \simeq \left(\frac{2\pi i}{\ln w}\right) 2r^{1/2}\cos\tfrac{1}{2}(\sigma+\theta_j-\theta_i) \,. \tag{2.12}$$

In the same limit the G's of (A.14) and (A.15) behave as

$$G(\rho_i/\rho_j, w) = \left(\frac{2\pi i}{\ln w}\right)(-\tfrac{1}{2}\cot\tfrac{1}{2}(\theta_i-\theta_j)) \,, \tag{2.13}$$

$$G_{\mathrm{T}}(\rho_i/\rho_j, w) = \left(\frac{2\pi i}{\ln w}\right)(-2r\sin(\theta_i-\theta_j+\sigma)) \,. \tag{2.14}$$

The partition function in (A.16) satisfies

$$\prod_{n=0}^{\infty}(1+w^{n+1/2})^D = \left(\frac{r^2}{w}\right)^{-D/48} \prod_{n=0}^{\infty}(1+r^{2n+1})^D \,. \tag{2.15}$$

As r tends toward zero, terms in G_{T} will be negligible compared to terms in G. Similarly χ_{T} would be negligible compared to χ were it not for the fact that terms in χ_{T} always multiply large s-channel variables whereas terms in χ multiply t-channel variables.

The two kinematical regions of present interest are the fragmentation region $\alpha(t_1), \alpha(t_2), \alpha(s_2) << \alpha(s_1) \lesssim \alpha(s)$ and the double-Regge region $\alpha(t_1), \alpha(t_2) << \alpha(s_1)$, $\alpha(s_2) << \alpha(s)$. Let us therefore neglect $\alpha(t_1)$, $\alpha(t_2)$ and $\alpha(s_2)$ in (A.23) and put $\alpha(s_1) = \kappa\alpha(s)$. The spin factor (A.23) then considerably simplifies:

$$\left(\frac{r^2}{w}\right)^{D/48} T_b = \epsilon \cdot k_5 \alpha^2(s)\{\kappa(1-\kappa)\chi(\rho_5/\rho_4)[\chi_T(\rho_3)\chi_T(\rho_4/\rho_2)$$
$$- \chi_T(\rho_3/\rho_2)\chi_T(\rho_4)] + iG(\rho_4/\rho_5)\kappa[\chi_T(\rho_3/\rho_2)\chi_T(\rho_5)$$
$$- \chi_T(\rho_5/\rho_2)\chi_T(\rho_3)]\} - \epsilon \cdot k_3 \alpha(s)^2 \{(1-\kappa)\chi(\rho_4/\rho_2)$$
$$\times [\chi_T(\rho_4/\rho_2)\chi_T(\rho_5) - \chi_T(\rho_4)\chi_T(\rho_5/\rho_2)]$$
$$- iG(\rho_4/\rho_3)\kappa[\chi_T(\rho_3/\rho_2)\chi(\rho_5) - \chi_T(\rho_5/\rho_2)\chi_T(\rho_3)]\} \,. \tag{2.16}$$

Using (2.11) to (2.15) we have in the limit of small r

$$T_b = \left(\frac{r^2}{w}\right)^{-D/48} \left(\frac{2\pi}{\ln w}\right)^3 2r\alpha^2(s) \sin \tfrac{1}{2}\theta_2 \left\{ \frac{\epsilon \cdot k_5 \kappa}{\sin \frac{1}{2}(\theta_5 - \theta_4)} \right.$$
$$\times (\sin \tfrac{1}{2}\theta_5 \cos \tfrac{1}{2}(\theta_5 - \theta_4) - (1-\kappa) \sin \tfrac{1}{2}\theta_4) - \frac{\epsilon \cdot k_3}{\sin \frac{1}{2}\theta_4}$$
$$\left. \times (\kappa \sin \tfrac{1}{2}\theta_5 \cos \tfrac{1}{2}\theta_4 + (1-\kappa) \sin \tfrac{1}{2}(\theta_5 - \theta_4)) \right\} . \tag{2.17}$$

Thus in the large-s limit there are no ρ-meson spin correlations across the large rapidity gap spanned by the pomeron. Such correlations are carried by the pomeron daughter trajectories and could be studied by developing the next order in r. Moreover, at the threshold of the final dissociated $\rho\pi$ system, $\epsilon \cdot k_3 \simeq 0$ and the amplitude is totally aligned along k_{5_μ}. Since at this threshold the ρ and $\rho\pi$ helicities are identical, the pomeron trajectory is expected to conserve t-channel helicity to leading order in $\alpha(s)$ in the reaction $\pi\pi \to (\rho\pi)\pi$ at enhancements near the $\rho\pi$ threshold. Factors in k_{3_μ} break this conservation when the energy s_2 increases, thus predicting a breaking all the stronger as the mass of the produced system is heavy.

We now have the spin factor in a concise form for insertion into the integrand of (2.10). Since the integral is dominated for large s by $r \simeq 0$ we can without error scale out the $\sin \frac{1}{2}\theta_2$ factor in the exponentials by writing $r = \frac{1}{2}r' \sin \frac{1}{2}\theta_2$. The θ_2 integral can then be done trivially and gives the pomeron form factor to the target (pion) state:

$$\int_0^{2\pi} d\theta_2 (2 \sin \tfrac{1}{2}\theta_2)^{\alpha_P(t_1) - \alpha_f(t_1) - 1} \equiv f_P(t_1)$$
$$= 2^{\alpha_P(t_1) - \alpha_f(t_1)} B[\tfrac{1}{2}(\alpha_P(t_1) - \alpha_f(t_1)), \tfrac{1}{2}] \,. \tag{2.18}$$

In the remaining integral we can simplify by putting $\theta = \frac{1}{2}\theta_5$ and $\Phi = \frac{1}{2}\theta_4$:

$$A_5 = g^5 f_{\rm P}(t_1)\, 2^{-\alpha_\pi(t_2)-\alpha_\pi(s_2)-\alpha_\rho(u_2)+2}\alpha(s)^2 \int \mathrm{d}r\, r^{1-\alpha_{\rm P}(t_1)} \int_0^{2\pi} \mathrm{d}\gamma \int_0^{\pi} \mathrm{d}\theta$$

$$\times \int_0^{\theta} \mathrm{d}\Phi\, (\sin(\theta-\Phi))^{-\alpha_\pi(t_2)} (\sin\theta)^{-\alpha_\rho(u_2)} (\sin\Phi)^{-\alpha_\pi(s_2)}$$

$$\times \exp\{-4r[\alpha(s)\sin(\theta-\Phi)\cos(\gamma+\theta) + \alpha(s_1)\sin\Phi\cos\gamma]\}$$

$$\times \left\{ \frac{\epsilon\cdot k_5\kappa}{\sin(\theta-\Phi)} [(1-\kappa)\sin\Phi - \sin\theta\cos(\theta-\Phi)] \right.$$

$$\left. + \frac{\epsilon\cdot k_3}{\sin\Phi} [\kappa\sin\theta\cos\Phi + (1-\kappa)\sin(\theta-\Phi)] \right\} . \tag{2.19}$$

3. The single-Regge limit

In this section we wish to explore the behavior of (2.19) in the region of large $\alpha(s_1)$ and $\alpha(s)$ but with s_2 near the $\pi\rho$ threshold. This is the region of interest for the Deck effect.

If we put $\gamma = \gamma' - \theta$ (afterwards dropping the prime) we can write

$$A_5 = -g^5 f_{\rm P}(t_1)\, 2^{2-\alpha_{\rm f}(t_1)}\alpha(s)^2 \int \mathrm{d}r\, r^{1-\alpha_{\rm P}(t_1)} \int_0^{2\pi} \mathrm{d}\gamma \int_0^{\pi} \mathrm{d}\theta \int_0^{\theta} \mathrm{d}\Phi$$

$$\times (\sin(\theta-\Phi))^{-\alpha_\pi(t_2)} (\sin\theta)^{-\alpha_\rho(u_2)} (\sin\Phi)^{-\alpha_\pi(s_2)}$$

$$\times \exp\{-4r\alpha(s_1)[\sin\Phi\cos(\gamma-\theta) + \kappa^{-1}\sin(\theta-\Phi)\cos\gamma]\}$$

$$\times \left\{ \epsilon\cdot k_5\kappa^2 \frac{\sin\Phi}{\sin(\theta-\Phi)} - (\kappa\epsilon\cdot k_5 + (1-\kappa)\epsilon\cdot k_3) \right.$$

$$\left. \times \frac{\sin(\theta-\Phi)}{\sin\Phi} + \kappa\epsilon\cdot(k_5-k_3)\sin\theta\cot\Phi \right\} . \tag{3.1}$$

We can simplify the nomenclature, putting $\alpha_{\rm P}(t_1) = \alpha_{\rm P}$, $\alpha_{\rm f}(t_1) = \alpha_{\rm f}$, $\alpha_\pi(t_2) = \alpha_2$, $\alpha_\pi(s_2) = \alpha(s_2)$, $\alpha_\rho(u_2) = \alpha(u)$. The kinematics is such that $\alpha_2 + \alpha(s_2) + \alpha(u) = \alpha_{\rm f}$.

On the second exponential in (3.1) let us use the Mellin-Barnes transform

$$\exp\{-4r\alpha(s_1)\kappa^{-1}\sin(\theta-\Phi)\cos\gamma\} = \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}n}{2\pi i}\,\Gamma(-n) \times [4r\alpha(s_1)\kappa^{-1}\sin(\theta-\Phi)\cos\gamma]^n\,. \tag{3.2}$$

In the limit of $\alpha(s_1)$ becoming asymptotic along the imaginary axis we can perform the r integration:

$$\begin{aligned} A_5 = &-g^5 f_{\mathrm{P}}(t_1)\, 2^{2-\alpha_{\mathrm{f}}}\,\alpha(s)^2\,[4\alpha(s_1)]^{\alpha_{\mathrm{P}}-2} \\ &\times \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}n}{2\pi i}\,\Gamma(-n)\,\Gamma(n+2-\alpha_{\mathrm{P}})\kappa^{-n} \int_0^{2\pi}\mathrm{d}\gamma \int_0^{\pi}\mathrm{d}\theta \int_0^{\theta}\mathrm{d}\Phi \\ &\times (\sin\Phi\cos(\gamma-\theta))^{\alpha_{\mathrm{P}}-2-n}\,[\sin(\theta-\Phi)\cos\gamma]^n(\sin(\theta-\Phi))^{-\alpha_2} \\ &\times (\sin\theta)^{-\alpha(u)}(\sin\Phi)^{-\alpha(s_2)}\{\quad\}\,. \end{aligned} \tag{3.3}$$

The curly bracket in (3.3) is the same as in (3.1).

The next step is the evaluation of the integral over γ

$$G(\theta) = \int_0^{2\pi} \mathrm{d}\gamma\,\cos\gamma^n(\cos(\gamma-\theta))^{\alpha_{\mathrm{P}}-2-n}\,. \tag{3.4}$$

This has a simple expression in terms of a simpler integral

$$H(a,b,\theta) = \int_{\theta}^{\pi} \mathrm{d}\gamma\,(\sin\gamma)^a(\sin(\gamma-\theta))^b\,. \tag{3.5}$$

The solution of this simpler integral is

$$\begin{aligned} H(a,b,\theta) = &(2\cos\tfrac{1}{2}\theta)^{a+b+1}\,B(a+1,b+1) \\ &\times {}_2F_1(\tfrac{1}{2}(1+a-b), \tfrac{1}{2}(1+b-a); \tfrac{1}{2}(a+b+3); \cos^2\tfrac{1}{2}\theta)\,. \end{aligned} \tag{3.6}$$

Paying careful attention to phases one finds

$$\begin{aligned} G(\theta) = &(1+\mathrm{e}^{-i\pi(\alpha_{\mathrm{P}}-2)})\,H(n,\alpha_{\mathrm{P}}-2-n,\theta) \\ &+(\mathrm{e}^{-i\pi n}+\mathrm{e}^{-i\pi(\alpha_{\mathrm{P}}-2-n)})\,H(n,\alpha_{\mathrm{P}}-2-n,\pi-\theta)\,. \end{aligned} \tag{3.7}$$

Using various hypergeometric function identities one can show that

$$G(\theta) = \frac{-2\pi^{3/2} e^{-i\pi\alpha_P/2}}{\Gamma[\frac{1}{2}\alpha_P]\Gamma[\frac{1}{2}(3-\alpha_P)]} \, {}_2F_1(2-\alpha_P+n, -n; \tfrac{1}{2}(3-\alpha_P); \sin^2 \tfrac{1}{2}\theta) \,. \quad (3.8)$$

Clearly the Φ integrals in (3.3) are also of the form (3.5):

$$F(\theta) = \int_0^\theta d\Phi \, (\sin \Phi)^{\alpha_P - 2 - \alpha(s_2) - n} \, (\sin(\theta - \Phi))^{n - \alpha_2}$$

$$\times \left\{ \epsilon \cdot k_5 \frac{\kappa^2 \sin \Phi}{\sin(\theta - \Phi)} - (\kappa\epsilon \cdot k_5 + (1-\kappa)\epsilon \cdot k_3) \right.$$

$$\left. \times \frac{\sin(\theta - \Phi)}{\sin \Phi} + \kappa \sin \theta \cot \Phi \epsilon \cdot (k_5 - k_3) \right\} . \quad (3.9)$$

One finds in fact, after factorizing some common Γ functions,

$$F(\theta) = (2 \sin \tfrac{1}{2}\theta)^{2\gamma - 2} \frac{\Gamma(\gamma + \alpha - 2)\Gamma(\gamma - \alpha + 1)}{\Gamma(2\gamma + 1)} \hat{F}(\theta) \quad (3.10)$$

with

$$\hat{F}(\theta) = \epsilon \cdot k_5 \kappa^2 (2 \sin \tfrac{1}{2}\theta)^2 (\gamma + \alpha - 2)(\gamma + \alpha - 1) \, {}_2F_1(\alpha, 1-\alpha; 1+\gamma; \sin^2 \tfrac{1}{2}\theta)$$

$$- (\kappa\epsilon \cdot k_5 + (1-\kappa)\epsilon \cdot k_3)(2 \sin^2 \tfrac{1}{2}\theta)^2 (\gamma - \alpha + 1)(\gamma - \alpha + 2)$$

$$\times {}_2F_1(\alpha - 2, 3 - \alpha; 1 + \gamma; \sin^2 \tfrac{1}{2}\theta) + 2\kappa \sin^2 \theta$$

$$\times \epsilon \cdot (k_5 - k_3) \gamma(\gamma - \alpha + 1) \, {}_2F_1(\alpha, 1 - \alpha; \gamma; \sin^2 \tfrac{1}{2}\theta) \,, \quad (3.11)$$

where to shorten the formulae we have put

$$\alpha \equiv \tfrac{1}{2}(\alpha_P - \alpha(s_2) + \alpha_2 + 1) - n \,, \quad (3.12)$$

$$\gamma \equiv \tfrac{1}{2}(\alpha_P - \alpha(s_2) - \alpha_2 - 1). \quad (3.13)$$

In substituting (3.10) and (3.8) into (3.3) it is convenient to define

$$\Gamma\begin{pmatrix} a_1, & a_2, & a_3 & \ldots & a_p \\ b_1, & b_2, & & \ldots & b_q \end{pmatrix} = [\prod_{i=1}^{p} \Gamma(a_i)] \, [\prod_{j=1}^{q} \Gamma(b_j)^{-1}] \,. \quad (3.14)$$

We then have

$$A_5 = g^5 f_P(t_1) 2^{-\alpha_f - 1} \pi^{3/2} [-4i\alpha(s_1)]^{\alpha_P} \kappa^{-2} \int_{-i\infty}^{+i\infty} \frac{dn}{2\pi i} \kappa^{-n}$$

$$\times\, \Gamma\begin{pmatrix} n+2-\alpha_{\rm P},\ n-\alpha_2,\ -n,\ \alpha_{\rm P}-\alpha(s_2)-2-n \\ \frac{1}{2}\alpha_{\rm P},\ \frac{1}{2}(3-\alpha_{\rm P}),\ \alpha_{\rm P}-\alpha(s_2)-\alpha_2 \end{pmatrix} R(n)\,, \tag{3.15}$$

where

$$R(n) = \int_0^\pi \mathrm{d}\theta\, (2\sin\tfrac{1}{2}\theta)^{\alpha_{\rm P}-\alpha_{\rm f}-3}$$
$$\times\, {}_2F_1(2-\alpha_{\rm P}+n, -n; \tfrac{1}{2}(3-\alpha_{\rm P}); \sin^2\tfrac{1}{2}\theta)\hat{F}(\theta)\,. \tag{3.16}$$

The important features of $R(n)$ are that it has no poles in the finite n plane and its behavior at asymptotic n is such that, since κ is always <1, the contour in (3.15) can be closed to the left.

$$\begin{aligned} A_5 &= g^5 f_{\rm P}(t_1)\, 2^{-\alpha_{\rm f}-1}\pi^{3/2}\,[-4i\alpha(s_1)]^{\alpha_{\rm P}}\,\kappa^{-2} \\ &\times\, \Gamma^{-1}(\tfrac{1}{2}\alpha_{\rm P}, \tfrac{1}{2}(3-\alpha_{\rm P}), \alpha_{\rm P}-\alpha(u)-\alpha_2+1)\sum_{k=0}^{\infty}\kappa^k\frac{(-1)^k}{k!} \\ &\times\, \{\Gamma(\alpha_{\rm P}-\alpha_2-k-2, 2-\alpha_{\rm P}+k, k-\alpha(s_2))\,\kappa^{2-\alpha_{\rm P}}R(\alpha_{\rm P}-2-k) \\ &+ \Gamma(\alpha_2-\alpha_{\rm P}+2-k, k-\alpha_2, \alpha_{\rm P}-\alpha(s_2)-\alpha_2-2+k)\,\kappa^{-\alpha_2}R(\alpha_2-k)\}\,. \end{aligned} \tag{3.17}$$

Using the formulae given in appendix B the values of the function R appearing in (3.17) can be determined analytically in terms of a finite sum of ratios of gamma functions. In this form the amplitude is amenable to a numerical study of its phenomenological properties. This together with the appropriate phase-space considerations will be discussed in sect. 5.

In order to understand some essential characteristics we now consider the dominant contributions for the limiting value of κ : $\kappa \to 0$ and near the forward directions $t_1 \approx 0$, $t_2 \approx 0$.

The analytic structure of (3.17) is similar to that found in the Born term. Each of the two terms has unphysical poles at integral values of $\alpha_{\rm P}(t_1) - \alpha_\pi(t_2)$. Only in the sum do these poles cancel. This cancellation can be shown independent of the structure of $R(n)$. However, if one attempts, for small κ, to truncate the series at any finite k there will be uncancelled poles somewhere in the physical region. This situation presents a challenge to the phenomenologist which has not been adequately studied even in the simpler case of the Born term. If one restricts one's attention however to a small region in $\alpha_{\rm P} - \alpha_2$ the amplitude is well-represented for small κ by a few intelligently chosen terms in k. The amplitude of course is strongly damped in t_1 because of the pomeron Regge behavior. If one therefore restricts one's attention to the region near $\alpha_{\rm P}(t_1) - \alpha_\pi(t_2) \simeq 1$ (e.g. $t_1, t_2 \approx 0$ in the case of physical intercepts), one can see that the dominant terms for small κ are those proportional to $R(\alpha_2)$,

$R(\alpha_2 - 1)$ and $R(\alpha_P - 2)$. *A priori* the leading terms as $\kappa \to 0$ are given by $R(\alpha_P - 2)$ and $R(\alpha_2 - 1)$ since they provide dominant behaviors in $\kappa^{-\alpha_P+2}$ and $\kappa^{-\alpha_2+1}$, respectively:

$$A_5(\alpha_P - 2)$$

$$= C\kappa^{-\alpha_P}\Gamma\begin{pmatrix} \frac{1}{2}(\alpha_P - \alpha_f),\ 1 - \frac{1}{2}\alpha_P,\ \frac{1}{2}(\alpha_P - \alpha_2) - 1,\ -\frac{1}{2}\alpha(s_2),\ \frac{1}{2}(1 - \alpha(u)) \\ \frac{1}{2}(\alpha_P + 1 - \alpha(u) - \alpha_2),\ \frac{1}{2}\alpha_P,\ \frac{1}{2}(\alpha_P - \alpha_2 - \alpha(s_2)),\ \frac{1}{2}(3 - \alpha(u) - \alpha(s_2)) \end{pmatrix}$$

$$\times \{\kappa\alpha(s_2)[(\alpha_P - \alpha_2 - 2)\,\epsilon\cdot(k_5 - k_3) - (\alpha_P - \alpha(u) - \alpha_2 - 1)\kappa\,\epsilon\cdot k_5]$$

$$- (1 - \alpha(u) - \alpha(s_2))(\alpha_P - \alpha_2 - 2)\,\epsilon\cdot k_3\}\ , \tag{3.18}$$

$$A_5(\alpha_2 - 1) = 4C\kappa^{-\alpha_2}\Gamma\begin{pmatrix} \frac{1}{2}(\alpha_P - \alpha_f),\ \alpha_P - \frac{1}{2}(\alpha_f + 1),\ \frac{1}{2}(\alpha_2 - \alpha_P) + 1,\ 1 - \frac{1}{2}\alpha_2 \\ \alpha_P + \frac{1}{2}(1 - \alpha_2 - \alpha_f),\ \frac{1}{2}\alpha_P,\ \frac{1}{2}(\alpha_P - \alpha_f + \alpha_2 + 1) \end{pmatrix}$$

$$\times \{\kappa\,\epsilon\cdot k_5(\alpha_P - \alpha(s_2) - \alpha_2) - \epsilon\cdot(k_5 - k_3)\}\ , \tag{3.19}$$

where C is the common factor

$$C = 2^{-\alpha_f - 3} g^5 \pi^{3/2} f_P(t_1)(-2i\alpha(s_1))^{\alpha_P}\ , \tag{3.20}$$

and $A_5(y)$ is the contribution to the amplitude given by the pole at $n = y$ in (3.15).

The form (3.18) has no t_2 channel resonance poles (no pion pole in particular) possesses singularities in $\alpha(s_2)$ and $\alpha(u)$ at even and odd integers respectively, displays typical factors of an f-dominated pomeron-reggeon-pion vertex. Moreover, at a singularity in $\alpha(u)$ (or $\alpha(s_2)$) all resonance poles in $\alpha(s_2)$ (or $\alpha(u)$) disappear thanks to $\Gamma^{-1}[\frac{1}{2}(3 - \alpha(u) - \alpha(s_2))]$. Thus the Steinmann relations [10] forbidding simultaneous discontinuities in overlapping energy variables are obeyed. The form (3.19) has only singularities at even integers of α_2. Only the term $R(\alpha_2)$ contains the pion pole in α_2 and therefore for the present we restrict our attention there:

$$A_5(\alpha_2) = 4C\kappa^{-\alpha_2}\epsilon\cdot k_5$$

$$\times\Gamma\begin{pmatrix} \frac{1}{2}(\alpha_P - \alpha_f),\ \alpha_P - \frac{1}{2}(\alpha_f + 1),\ \frac{1}{2}(\alpha_2 - \alpha_P) + 1,\ -\frac{1}{2}\alpha_2 \\ \alpha_P - \frac{1}{2}(\alpha_f + \alpha_2 + 1),\ \frac{1}{2}(1 + \alpha_P - \alpha_f + \alpha_2),\ \frac{1}{2}\alpha_P \end{pmatrix}\ . \tag{3.21}$$

This contribution has no discontinuities in overlapping energy invariants, in particular no resonances in the s_2 channel, and thus appears to be an interesting candidate for a Deck effect like contribution in the dual framework, since this amplitude is diffractive, includes the pion singularity and totally lacks resonances in the s_2 channel.

Two further interesting points must be mentioned. This part of the amplitude is t-channel helicity conserving for the $\pi\rho$ system and becomes relatively more important for small κ at the threshold production of this system since the leading behavior of $R(\alpha_P - 2)$ is now $\kappa^{1-\alpha_P}$.

4. Double-Regge limit

For Born-term reggeons a structure of the five-point amplitude B_5 consistent with the Steinmann relations emerges naturally [11]. The particle-double-Regge vertex breaks up into two components corresponding to two possible combinations of singularities in non-overlapping variables:

$$B_5^{RR} = \beta_1(t_1)\beta_2(t_2)\alpha^{\alpha_1}(s_1)\alpha^{\alpha_2}(s_2)[\xi_1\xi_{21}\eta^{\alpha_1}V(\alpha_1,\alpha_2;\eta) + 1 \leftrightarrow 2] \tag{4.1}$$

where

$$\xi_i = \tau_i + e^{-i\pi\alpha_i}, \qquad \xi_{ij} = \tau_i\tau_j + e^{-i\pi(\alpha_i-\alpha_j)}, \qquad \eta = \frac{\alpha(s)}{\alpha(s_1)\,\alpha(s_2)},$$

$$V(\alpha_1,\alpha_2;\eta) = \sum_{n=0}^{\infty} \Gamma\begin{pmatrix} -\alpha_1+n,\ \alpha_1-\alpha_2-n \\ n+1 \end{pmatrix}\eta^{-n}. \tag{4.2}$$

τ_i is the signature of the trajectory α_i. In expression (4.1) it is worthwhile to note the divergence of signature factors from a simple factorization hypothesis, and the allocation of poles between the two parts (α_1 or α_2 integer, respectively). This structure has been of great importance in the discussion of decoupling theorems [12] and possesses interesting phenomenological consequences [13,14].

Now our aim is to discuss the analytic structure of the pomeron-particle-reggeon vertex; avoiding a complete but quite tedious calculation we prefer to limit ourselves to the study of leading contributions as $\eta \to \infty$. For this purpose it is sufficient to take first the limit $\alpha(s_2) \to \infty$ with η and t_2 fixed in eq. (3.17). The leading term is obtained from $R(\alpha_P - 2)$:

$$B^{PR}(\alpha_P - 2) = -C'(2\eta)^{\alpha_P}2^{\alpha_P-\alpha_2}e^{-i\pi\alpha_2/2}\,\epsilon\cdot k_3$$
$$\times\,\Gamma\begin{pmatrix} 1-\frac{1}{2}\alpha_P,\ \frac{1}{2}(\alpha_P-\alpha_2),\ \frac{1}{2}(\alpha_P-\alpha_f) \\ \frac{1}{2}\alpha_P,\ \frac{1}{2}(\alpha_2-\alpha_f+1) \end{pmatrix}, \tag{4.3}$$

$$C' = 2^{-1-\alpha_f}\pi^{3/2}f_P(t_1)\,\alpha^{\alpha_P}(s_1)\,\alpha^{\alpha_2}(s_2)\,; \tag{4.4}$$

and the term required by analyticity in order to compensate the pole at $\alpha_P = \alpha_2$ is given by $R(\alpha_2 - 2)$:

$$B^{\mathrm{PR}}(\alpha_2 - 2) = -C'(2\eta)^{\alpha_2} \mathrm{e}^{-i\pi\alpha_{\mathrm{P}}/2} \epsilon \cdot k_3 \times \Gamma\begin{pmatrix} 1 - \frac{1}{2}\alpha_2, \ \frac{1}{2}(\alpha_2 - \alpha_{\mathrm{P}}), \ \alpha_{\mathrm{P}} - \frac{1}{2}(\alpha_{\mathrm{f}} + 1), \ \frac{1}{2}(\alpha_{\mathrm{P}} - \alpha_{\mathrm{f}}) \\ \frac{1}{2}\alpha_{\mathrm{P}}, \ \frac{1}{2}(\alpha_{\mathrm{P}} + \alpha_2 - \alpha_{\mathrm{f}} - 1), \ \alpha_{\mathrm{P}} + \frac{1}{2}(1 - \alpha_{\mathrm{f}} - \alpha_2) \end{pmatrix} . \tag{4.5}$$

However, this last contribution does not possess the pion pole and another very important contribution in this limit is

$$B^{\mathrm{PR}}(\alpha_2) = C'(2\eta)^{\alpha_2} \mathrm{e}^{-i\pi\alpha_{\mathrm{P}}/2} \epsilon \cdot k_5 \times \Gamma\begin{pmatrix} -\frac{1}{2}\alpha_2, \ \frac{1}{2}(\alpha_2 - \alpha_{\mathrm{P}}) + 1, \ \alpha_{\mathrm{P}} - \frac{1}{2}(\alpha_{\mathrm{f}} + 1), \ \frac{1}{2}(\alpha_{\mathrm{P}} - \alpha_{\mathrm{f}}) \\ \frac{1}{2}\alpha_{\mathrm{P}}, \ \frac{1}{2}(\alpha_{\mathrm{P}} + \alpha_2 - \alpha_{\mathrm{f}} + 1), \ \alpha_{\mathrm{P}} - \frac{1}{2}(1 + \alpha_{\mathrm{f}} + \alpha_2) \end{pmatrix} . \tag{4.6}$$

Some comments are in order concerning these expressions.

(i) Since Γ functions of the form $\Gamma[\frac{1}{2}(k - \alpha_2)]$ never appear in (4.5) or (4.6) and since (4.3) and (4.5) display the factors $\exp[-\frac{1}{2}i\pi\alpha_2]\,\Gamma[1 - \frac{1}{2}\alpha_{\mathrm{P}}]$ and $\exp[-\frac{1}{2}i\pi\alpha_{\mathrm{P}}] \times \Gamma[1 - \frac{1}{2}\alpha_2]$, respectively, the same signature factor $\xi_i\xi_{ji}$ occur in the pomeron-particle-reggeon vertex and in the particle-double-regge vertex (*cf.* eq. (4.1)). Moreover, the pomeron can be coupled only to a reggeon of positive signature. This last result is not surprising since only resonances at even integers are present in the α_2 channel.

(ii) The spin dependence induced by the leading term (4.3) is very different from the spin dependence given by the pion pole (eq. (4.6)).

(iii) Unlike in (4.1), there is no symmetry between (4.3) and (4.5). This is a direct consequence of the distinct character of the pomeron compared to reggeons in the dual models.

5. Numerical results and discussion

We present now a detailed numerical evaluation of the amplitude with appropriate phase-space considerations and comparison with experimental features. Both the double-Regge limit (DR) and the single-Regge formulation (SR) are studied. Although the DR limit is *a priori* inadequate (since in the domain of interest one of the sub-energies is barely above threshold) we think that this formulation merits some consideration since it avoids the presence of poles (resonances) in the physical region, a defect inherent in the SR formulation and more generally in all Veneziano-like formulae.

Moreover, from well-known duality phenomenology, the DR description is believed to give on the average a good representation of resonance effects.

We are first confronted with the difficulty associated with the presence of unphysical poles in our amplitudes at integral values of $\alpha_P(t_1) - \alpha_\pi(t_2)$.

We have verified that in the kinematic domain of interest: $|t_1| \lesssim 0.5\ (\mathrm{GeV})^2$, $M_{\rho\pi} \lesssim 1.6$ GeV, and for our choice of trajectories $\alpha_P(t_1)$ and $\alpha_\pi(t_2)$, a unique singularity appears for $\alpha_P(t_1) - \alpha_\pi(t_2) = 2$. In the SR limit this spurious pole is automatically cancelled order-by-order in the expansion in k (eq. (3.17)). In the DR limit our formulation includes the dominant term (4.3) in energy, however strongly suppressed by the kinematical factor $\epsilon \cdot k_3$, and the term (4.6) exhibiting the pion singularity. This last term having a spurious pole at $\alpha_P(t_1) - \alpha_\pi(t_2) = 2$, we must add an extra piece cancelling this unwanted singularity. This piece is given by a non-leading order contribution in η when the limit $\alpha(s_2) \to \infty$ is taken in $A_5(\alpha_P - 2)$:

$$-C'(2\eta)^{\alpha_P - 2} 2^{\alpha_P - \alpha_2 - 2} \epsilon \cdot k_5\, \mathrm{e}^{-i\pi\alpha_2/2}$$

$$\times \Gamma\begin{pmatrix} 1 - \frac{1}{2}\alpha_P,\ \frac{1}{2}(\alpha_P - \alpha_f),\ \frac{1}{2}(\alpha_P - \alpha_2) - 1 \\ \frac{1}{2}\alpha_P,\ \frac{1}{2}(3 + \alpha_2 - \alpha_f) \end{pmatrix}.$$

Conventional phenomenological trajectories have been used:

$$\alpha_P(t_1) = 1 + \tfrac{1}{2}t_1, \qquad \alpha_f(t_1) = 0.5 + t_1, \qquad \alpha_\pi(t_2) = -\mu_\pi^2 + t_2 .$$

Furthermore, it was necessary to give a small imaginary part to the trajectory in

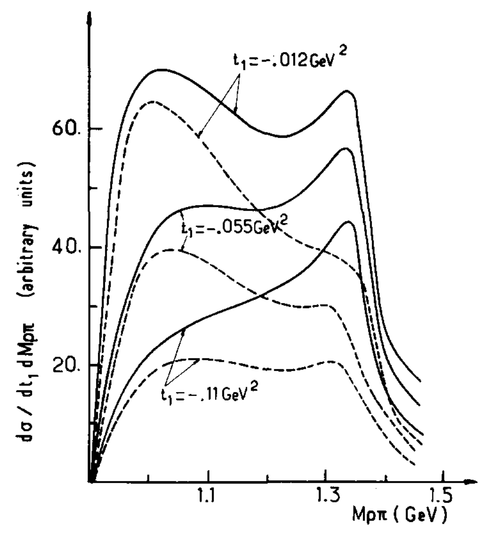

Fig. 3. Calculated $\rho\pi$ mass distribution in the single-Regge limit, for three values of the transfer momentum t_1 (−0.012, −0.055 and −0.11 GeV^2). Solid curves: full amplitude; dashed curves: 1^+S, $M = 0$ contribution.

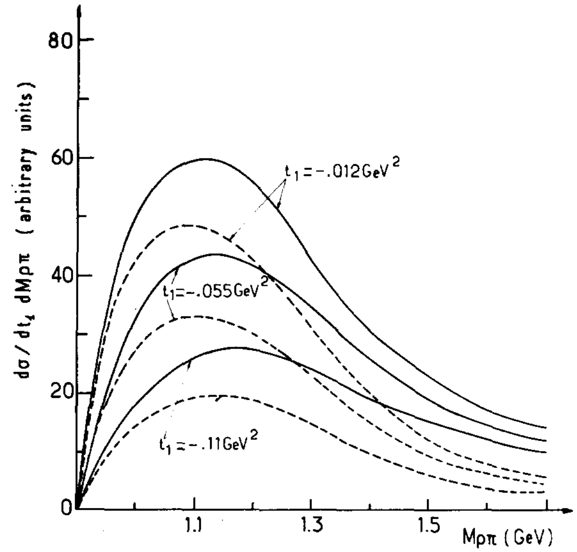

Fig. 4. Calculated $\rho\pi$ mass distribution in the double-Regge limit for three values of the transfer momentum t_1 (−0.012, −0.055 and −0.11 GeV^2). Solid curves: full amplitude; dashed curves: 1^+S, M = 0 contribution.

the production channel $\alpha(s_2)$:

$$\alpha(s_2) = s_0 + s_2 + i\lambda\sqrt{s_2 - (m_\rho + \mu_\pi)^2}\ ,$$

in order to add an *ad hoc* width to resonances. Thus, in the SR limit results depend on the parameters s_0 and λ which control positions and strengths of resonances. The choice adopted (s_0 = 0.137 GeV^2, λ = 0.1825 GeV) corresponds to a peak near 1.3 GeV slightly emerging from the non-resonant background as data show [15]. Finally, it is interesting to notice that the series (3.17) converges quite rapidly; the first three terms are sufficient to give a very satisfactory approximation in the restricted kinematic domain considered.

Our most significant results are depicted in figs. 3 to 6. The lab. momentum was arbitrarily taken to be 13 GeV/c and masses of the proton were assigned to particles 1 and 2 in order to recover a realistic kinematical situation. As a practical matter, our conclusions are almost insensitive to variations in energy or to this last assignment. We see in figs. 3 and 4 that both calculations furnish a strong enhancement in the invariant-mass distribution just above threshold. A partial-wave analysis allows us to show that the low-mass distribution is predominantly an S-wave $J^P = 1^+$ state. The method of partial-wave analysis is borrowed from Berger and Donohue [4], where technical details can be found. The partial waves were obtained in the $\rho\pi$ t-channel system of axes; J is the total spin of the $\rho\pi$ system and M is its projection

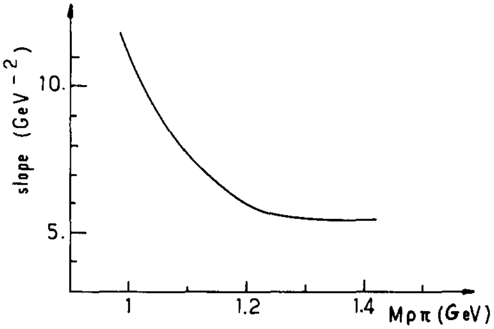

Fig. 5. Variation of the slope parameter as a function of the invariant mass $M_{\rho\pi}$.

along the t-channel z-axis. We have verified that amplitudes with $M > 0$ are very small, as expected since our model provides approximately t-channel helicity conservation, and we do not discuss them further.

The peak of the distribution occurs at $M_{\rho\pi} \simeq 1.1$ GeV for the DR formulae (fig. 4) in agreement with data whereas for the SR limit this peak is somewhat nearer to threshold but the exact position now depends on the strength given to the resonant part of the amplitude. In this case the $M = 0$, 1^+S contribution possesses a shoulder structure near the resonance region. This is due to a contamination of the resonating part of the amplitude which contributes not only to partial waves specific to the A_2 resonance but also to partial waves with smaller angular momentum. This is a well-known defect of Veneziano-like formulations. A detailed analysis of other partial waves would require one first to cure this disease.

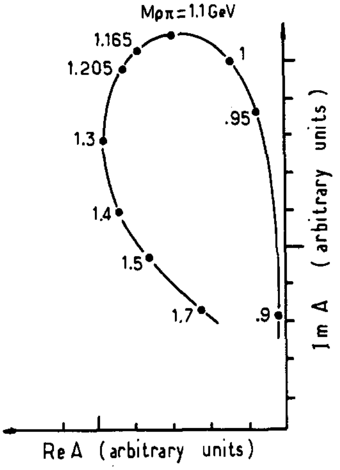

Fig. 6. The Argand diagram for the 1^+S, $M = 0$ partial wave as a function of $M_{\rho\pi}$. Units are arbitrary.

It is worthwhile to note that duality appears to work as expected: although the SR calculation seems to be a bit stronger than the DR results, the latter reproduce the SR ones both quantitatively and qualitatively since the order of magnitude of cross sections are the same, and since the strong enhancement near $M_{\rho\pi} = 1.1$ GeV is comparable in shape and strength.

In fig. 5 we have plotted the mass-slope correlation given by this model. The result compares quite favorably with the experimental slope [16] of 10.6 ± 1.2 GeV^{-2} in the A_1 region and of 5.8 ± 1.1 GeV^{-2} in the A_2 region.

Finally, in fig. 6 we present the Argand plot of the real *versus* the imaginary part of the partial wave $1^+\mathrm{S}$, $(M = 0)$ for $t_1 = -0.05$ GeV^2 in the DR case. Surprisingly, this resembles resonant behavior in two-body reactions since an anti-clockwise loop is described with strong variation near $M_{\rho\pi} \simeq 1.1$ GeV.

This result is very different from other calculations [17] based on a more naive double-Regge formulation where there appears to be no evidence for any resonant behavior in this diagram.

We have studied the leading contribution of a single dual pomeron to diffractive dissociation of a π meson into a $\rho\pi$ system in the framework of the Neveu-Schwarz model. It was found that the resulting amplitude satisfies asymptotically t-channel helicity conservation at the threshold of the $\rho\pi$ system, a result in nice agreement with data. In the single-Regge limit, for $\kappa \to 0$ two competing main contributions are obtained, one of them containing the pion pole, the other one having resonances in the $\rho\pi$ channel and the associated crossed channel.

The double-Regge limit allows one to compare the double reggeon-particle vertex, known for a long time, to the pomeron-reggeon-particle vertex. We find that the separation in two terms required by analyticity breaks the phase factors in the same manner for these two vertices.

Finally we have shown that the non-resonant mass spectrum predicted by this model, which respects analyticity properties (Steinmann relations) and satisfies requirement of duality, does not require a resonant A_1.

We take this opportunity to express our gratitude to J.T. Donohue for his interest in this work and for useful remarks. We are also grateful to him for communicating, and assistance with, his numerical program for projecting amplitudes in partial waves.

Appendix A

The trace in (2.6) factors into two terms, one being merely the trace over the orbital oscillators and the second being a trace over the fermionic oscillators of the NSM. Each of these is reducible to a vacuum expectation value,

$$\mathrm{Tr}\{w^{L_0}\Omega V_\pi(k_1, 1)\, V_\pi(k_2, \rho_2)\,\Omega V_\pi(k_3, \rho_3)\, V_\rho(k_4, \rho_4)\, V_\pi(k_5, \rho_5)\}$$
$$= T_a T_b\,, \tag{A.1}$$

$$T_a = \mathrm{Tr}\{w^{L_{0a}} :\mathrm{e}^{ik_1\cdot Q(-1)}: :\mathrm{e}^{ik_2\cdot Q(-\rho_2)}:$$

$$\times :\mathrm{e}^{ik_3\cdot Q(\rho_3)}: :\mathrm{e}^{ik_4\cdot Q(\rho_4)}: :\mathrm{e}^{ik_5\cdot Q(\rho_5)}:\}\,, \tag{A.2}$$

$$T_b = \mathrm{Tr}\{w^{L_{0b}} k_1 \cdot H(-1) k_2 \cdot H(-\rho_2) k_3 \cdot H(\rho_3)$$

$$\times [k_4 \cdot H(\rho_4) \epsilon \cdot H(\rho_4) + i \sum_{\substack{j=1\\ \neq 4}}^{5} k_j \cdot \epsilon\, G_{(\mathrm{T})}(\rho_4/\rho_j, w)] k_5 \cdot H(\rho_5)\,. \tag{A.3}$$

For i and j not separated by a twist

$$G_{(\mathrm{T})}(\rho_i/\rho_j, w) \equiv G(\rho_i/\rho_j, w)$$

$$= i\left\{-\frac{\ln(\rho_i/\rho_j)}{\ln w} + i\rho_i \frac{\mathrm{d}}{\mathrm{d}\rho_i} \ln \theta_1\left(\frac{\ln(\rho_i/\rho_j)}{2\pi i} \Big/ \frac{\ln w}{2\pi i}\right)\right\}, \tag{A.4}$$

whereas for i and j separated by a twist

$$G_{(\mathrm{T})}(\rho_i/\rho_j, w) = G_{\mathrm{T}}(\rho_i/\rho_j, w)$$

$$= i\left\{-\frac{\ln(\rho_i/\rho_j)}{\ln w} + i\rho_i \frac{\mathrm{d}}{\mathrm{d}\rho_i} \ln \theta_2\left(\frac{\ln(\rho_i/\rho_j)}{2\pi i} \Big/ \frac{\ln w}{2\pi i}\right)\right\}. \tag{A.5}$$

T_a is the same trace that occurs in the conventional Veneziano model:

$$T_a = \left(-\frac{2\pi}{\ln w}\right)^{D/2} \delta^D(\sum_i k_i) f(w)^{-D} \prod_{i<j} \Psi_{(\mathrm{T})}(\rho_j/\rho_i, w)^{k_i\cdot k_j}. \tag{A.6}$$

Here D is the dimensionality of space-time (10 for NSM),

$$f(w) = \prod_{n=1}^{\infty} (1 - w^n)\,, \tag{A.7}$$

and either

$$\Psi_{(\mathrm{T})}(\chi, w) = \Psi(\chi, w) = -2\pi i\left(\exp \frac{\ln^2\chi}{2\ln w}\right) \frac{\theta_1\left(\frac{\ln\chi}{2\pi i} \Big/ \frac{\ln w}{2\pi i}\right)}{\theta_1'\left(0 \Big/ \frac{\ln w}{2\pi i}\right)} \tag{A.8}$$

or

$$\Psi_{(\mathrm{T})}(\chi, w) = \Psi_{\mathrm{T}}(\chi, w) = 2\pi\left(\exp \frac{\ln^2\chi}{2\ln w}\right) \frac{\theta_2\left(\frac{\ln\chi}{2\pi i} \Big/ \frac{\ln w}{2\pi i}\right)}{\theta_1'\left(0 \Big/ \frac{\ln w}{2\pi i}\right)}\,, \tag{A.9}$$

depending on whether i and j are separated by a twist (A.8) or not (A.9).

In terms of the variables defined by (2.8a) and (2.8b)

$$\Psi(\rho_j/\rho_i) = -\frac{2\pi}{\ln r} \sin \tfrac{1}{2}(\theta_j - \theta_i) \prod_{n=1}^{\infty} \frac{1 - 2r^{2n}\cos(\theta_j - \theta_i) + r^{4n}}{(1 - r^{2n})^2} , \tag{A.10}$$

$$\Psi_{\mathrm{T}}(\rho_j/\rho_i) = -\frac{\pi}{\ln r} r^{-1/4} \prod_{n=1}^{\infty} \frac{1 - 2r^{2n-1}\cos(\theta_j - \theta_i) + r^{4n-2}}{(1 - r^{2n})^2} , \tag{A.11}$$

$$f(w) = i^{1/3}\left(\frac{\ln r}{\pi}\right)^{1/2} w^{-1/24} r^{1/12} f(r^2) , \tag{A.12}$$

$$S(w) = r \frac{f(r^2)^2}{\prod_{n=0}^{\infty} (1 + r^{2n+1})^2} , \tag{A.13}$$

$$G(\rho_i/\rho_j, w) = -\frac{2\pi i}{\ln w} \frac{\mathrm{d}}{\mathrm{d}\theta_i} \ln \theta_1\left(\frac{\theta_i - \theta_j}{2\pi} \Big/ \frac{\ln r^2}{2\pi i}\right) , \tag{A.14}$$

$$G_{\mathrm{T}}(\rho_i/\rho_j, w) = -\frac{2\pi i}{\ln w} \frac{\mathrm{d}}{\mathrm{d}\theta_i} \ln \theta_4\left(\frac{\theta_i - \theta_j}{2\pi i} \Big/ \frac{\ln r^2}{2\pi i}\right) . \tag{A.15}$$

The trace T_b of eq. (A.3) can be written in terms of a vacuum expectation value:

$$T_b = \prod_{n=0}^{\infty} (1 + w^{n+1/2})^D \langle 0| k_1 \cdot \hat{H}(-1) k_2 \cdot \hat{H}(-\rho_2) k_3 \cdot \hat{H}(\rho_3)$$

$$\times [k_4 \cdot \hat{H}(\rho_4) \epsilon \cdot \hat{H}(\rho_4) + i \sum_{\substack{j=1 \\ \neq 4}}^{5} \epsilon \cdot k_j G_{\mathrm{T}}(\rho_4/\rho_j, w)] k_5 \cdot \hat{H}(\rho_5)|0\rangle . \tag{A.16}$$

The $\hat{H}$ of (A.16) is the same operator as H of (2.2) except that the fermionic oscillators b_μ^n and b_μ^{n+} of the original H are replaced by

$$d_\mu^n = \frac{b_\mu^n}{1 + w^{n+1/2}} + b_\mu^{n'+} , \tag{A.17}$$

$$\overline{d_\mu^n} = b_\mu^{n+} + \frac{b_\mu^{n'} w^{n+1/2}}{1 + w^{n+1/2}} , \tag{A.18}$$

respectively.

A vacuum expectation value of an even number of $\hat{H}$'s is given by contracting them in pairs in all possible ways. That is:

$$\langle 0| \prod_{i=1}^{2N} h_i \cdot \hat{H}(\rho_i)|0\rangle = \sum_{\text{perms}} (-1)^P \prod_{j=1}^{N} h_{i_{2j-1}} \cdot h_{i_{2j}} \chi^{(1/2)}(\rho_{i_{2j}}/\rho_{i_{2j-1}}, w) .$$

The parity P of the permutation is the number of $\hat{H}$'s that are skipped over in pairing the $\hat{H}$'s two-by-two.

$$h_i \cdot h_j \, \chi^{(1/2)}(\rho_j/\rho_i, w) = \langle 0 | h_i \cdot \hat{H}(\rho_i) h_j \cdot \hat{H}(\rho_j) | 0 \rangle \,. \tag{A.20}$$

One finds before and after the Jacobi transformation (2.8a, b)

$$\chi^{(1/2)}(\rho_j/\rho_i, w) = \sum_{m=0}^{\infty} \frac{(\rho_j/\rho_i)^{m+1/2} + (w\rho_i/\rho_j)^{m+1/2}}{1 + w^{m+1/2}}$$

$$= \frac{2\pi i}{\ln w} \sum_{m=0}^{\infty} \frac{(\rho_j'/\rho_i')^{m+1/2} + (w'\rho_i'/\rho_j')^{m+1/2}}{1 + w'^{m+1/2}} \,. \tag{A.21}$$

If i and j are separated by a twist it is sometimes convenient to define a $\chi_{\mathrm{T}}^{(1/2)}$:

$$\chi_{\mathrm{T}}^{(1/2)}(\rho_j/\rho_i, w) = \chi^{(1/2)}(-\rho_j/\rho_i, w) = \frac{2\pi i}{\ln w} \chi^{(1/2)}(r\rho_j'/\rho_i', r^2) \,. \tag{A.22}$$

Applying (A.19) to (A.16) we have

$$\prod_{n=0}^{\infty} (1 + w^{n+1/2})^{-D} T_b = \epsilon \cdot k_5 \chi(\rho_5/\rho_4)[k_3 \cdot k_4 \, k_1 \cdot k_2 \chi(\rho_4/\rho_3) \chi(\rho_2)$$
$$+ k_2 \cdot k_3 \, k_1 \cdot k_4 \chi_{\mathrm{T}}(\rho_3/\rho_2) \chi_{\mathrm{T}}(\rho_4) - k_1 \cdot k_3 \, k_2 \cdot k_4 \chi_{\mathrm{T}}(\rho_3) \chi_{\mathrm{T}}(\rho_4/\rho_2)]$$
$$- \epsilon \cdot k_3 \chi(\rho_4/\rho_3)[k_4 \cdot k_5 \, k_1 \cdot k_2 \chi(\rho_5/\rho_4) \chi(\rho_2) + k_2 \cdot k_4 \, k_1 \cdot k_5$$
$$\times \chi_{\mathrm{T}}(\rho_4/\rho_2) \chi_{\mathrm{T}}(\rho_5) - k_1 \cdot k_4 \, k_2 \cdot k_5 \chi_{\mathrm{T}}(\rho_4) \chi_{\mathrm{T}}(\rho_5/\rho_2)] + \epsilon \cdot k_2$$
$$\times \chi_{\mathrm{T}}(\rho_4/\rho_2)[k_4 \cdot k_5 \, k_1 \cdot k_3 \chi(\rho_5/\rho_4) \chi_{\mathrm{T}}(\rho_3) + k_3 \cdot k_4 \, k_1 \cdot k_5 \chi(\rho_4/\rho_3)$$
$$\times \chi_{\mathrm{T}}(\rho_5) - k_3 \cdot k_5 \, k_1 \cdot k_4 \chi(\rho_5/\rho_3) \chi_{\mathrm{T}}(\rho_4)] - \epsilon \cdot k_1 \chi_{\mathrm{T}}(\rho_4)[k_4 \cdot k_5$$
$$\times k_2 \cdot k_3 \chi(\rho_5/\rho_4) \chi_{\mathrm{T}}(\rho_3/\rho_2) + k_3 \cdot k_4 \, k_2 \cdot k_5 \chi(\rho_4/\rho_3) \chi_{\mathrm{T}}(\rho_5/\rho_2)$$
$$- k_3 \cdot k_5 \, k_2 \cdot k_4 \chi(\rho_5/\rho_3) \chi_{\mathrm{T}}(\rho_4/\rho_2)] + i[\epsilon \cdot k_5 G(\rho_4/\rho_5) + \epsilon \cdot k_3 G(\rho_4/\rho_3)$$
$$+ \epsilon \cdot k_2 G_{\mathrm{T}}(\rho_4/\rho_2) + \epsilon \cdot k_1 G_{\mathrm{T}}(\rho_4)] \, [k_3 \cdot k_5 \, k_1 \cdot k_2 \chi(\rho_5/\rho_3) \chi(\rho_2)$$
$$+ k_2 \cdot k_3 \, k_1 \cdot k_5 \chi_{\mathrm{T}}(\rho_3/\rho_2) \chi_{\mathrm{T}}(\rho_5) - k_2 \cdot k_5 \, k_1 \cdot k_3 \chi(\rho_5/\rho_2) \chi_{\mathrm{T}}(\rho_3)] \,. \tag{A.23}$$

Appendix B

Here we derive some useful formulae allowing one to obtain the residues of poles in the n plane (3.17) and for arbitrary integer k. First we exploit a property of hypergeometric functions [18]:

$$F(a, b; c; x) = (1-x)^{c-a-b} F(c-a, c-b; c; x) \tag{B.1}$$

which enables us to rewrite (3.16) in the form

$$\begin{aligned} R(n) = \{ & \epsilon \cdot k_5\, \kappa^2(\alpha_{\mathrm{P}} - \alpha(s_2) - n - 2)(\alpha_{\mathrm{P}} - \alpha(s_2) - n - 1) \\ & \times G(2 - \alpha_{\mathrm{P}} + n, -n; n - \alpha_2, \alpha_{\mathrm{P}} - \alpha(s_2) - n; \tfrac{1}{2}(\alpha_{\mathrm{P}} - \alpha_{\mathrm{f}})) \\ & - [\kappa\epsilon \cdot k_5 + \epsilon \cdot k_3](n - \alpha_2)(n - \alpha_2 + 1) \\ & G(2 - \alpha_{\mathrm{P}} + n, -n; n - \alpha_2 + 2, \alpha_{\mathrm{P}} - \alpha(s_2) - n - 2; \tfrac{1}{2}(\alpha_{\mathrm{P}} - \alpha_{\mathrm{f}})) \\ & + \kappa\epsilon \cdot (k_5 - k_3)(n - \alpha_2)(\alpha_{\mathrm{P}} - \alpha(s_2) - \alpha_2 - 1) \\ & \times G(2 - \alpha_{\mathrm{P}} + n, -n; n - \alpha_2, \alpha_{\mathrm{P}} - \alpha(s_2) - n - 2; \tfrac{1}{2}(\alpha_{\mathrm{P}} - \alpha_{\mathrm{f}}))\} \;, \end{aligned} \tag{B.2}$$

where we have defined

$$\begin{aligned} G(2\alpha, 2\beta; 2\alpha', 2\beta'; b) = & \int_0^\pi \mathrm{d}\Phi\, (\sin\Phi)^{2b-1} \\ & \times F(2\alpha, 2\beta; \alpha + \beta + \tfrac{1}{2}; \sin^2 \tfrac{1}{2}\Phi)\, F(2\alpha', 2\beta'; \alpha' + \beta' + \tfrac{1}{2}; \cos^2 \tfrac{1}{2}\Phi) \,. \end{aligned} \tag{B.3}$$

Next we use a quadratic transformation of the hypergeometric function:

$$\begin{aligned} F(2a, 2b; a + b + \tfrac{1}{2}; \tfrac{1}{2}(1 \mp \sqrt{x})) = & \Gamma\begin{pmatrix} \tfrac{1}{2}, & a + b + \tfrac{1}{2} \\ a + \tfrac{1}{2}, & b + \tfrac{1}{2} \end{pmatrix} F(a, b; \tfrac{1}{2}; x) \\ & \pm \sqrt{x}\, \Gamma\begin{pmatrix} -\tfrac{1}{2}, & a + b + \tfrac{1}{2} \\ a, & b \end{pmatrix} F(a + \tfrac{1}{2}, b + \tfrac{1}{2}; \tfrac{3}{2}; x) \,, \end{aligned} \tag{B.4}$$

to find that (B.4) is

$$\begin{aligned} G(2\alpha, 2\beta; 2\alpha', 2\beta'; b) = & \pi\Gamma(\alpha + \beta + \tfrac{1}{2}, \alpha' + \beta' + \tfrac{1}{2}) \int_0^1 \mathrm{d}x\, x^{-1/2}(1-x)^{b-1} \\ & \times \left\{ \frac{F(\alpha, \beta; \tfrac{1}{2}; x)\, F(\alpha', \beta'; \tfrac{1}{2}; x)}{\Gamma(\alpha + \tfrac{1}{2}, \beta + \tfrac{1}{2}, \alpha' + \tfrac{1}{2}, \beta' + \tfrac{1}{2})} \right. \\ & \left. - 4x \frac{F(\alpha + \tfrac{1}{2}, \beta + \tfrac{1}{2}; \tfrac{3}{2}; x)\, F(\alpha' + \tfrac{1}{2}, \beta' + \tfrac{1}{2}; \tfrac{3}{2}; x)}{\Gamma(\alpha, \beta, \alpha', \beta')} \right\} \,. \end{aligned} \tag{B.5}$$

Then the problem of evaluating (3.17) is restricted to calculating the residues of $\Gamma(2\alpha, 2\beta, 2\alpha', 2\beta')\,G(2\alpha, 2\beta; 2\alpha', 2\beta'; b)$ when one of the arguments 2α, or $2\alpha'$ is equal to a negative integer. Let us consider for instance $2\alpha'$. Following the parity of $2\alpha'$ one term only of (B.5) gives a residue; one of the two hypergeometric function is reduced to a polynomial in x and the last step to be done is the calculation of integrals of the type

$$\int_0^1 \mathrm{d}x\, x^{a+k-1}(1-x)^{b-1}\, F(\alpha, \beta; a; x) = I_k(a, b; \alpha, \beta)\,, \tag{B.6}$$

which is a representation of the hypergeometric function ${}_3F_2$:

$$I_k(a, b; \alpha, \beta) = B(a+k, b)\; {}_3F_2(\alpha, \beta, a+k; a, a+b+k; 1)\,. \tag{B.7}$$

In order to exhibit clearly the analyticity properties of the result, it is convenient to use a generalization of Dixon's theorem [19] and rewrite

$$I_k(a, b; \alpha, \beta) = \Gamma\begin{pmatrix} a,\ b,\ a+b-\alpha-\beta \\ a+b-\beta, a+b-\alpha \end{pmatrix}$$

$$\times\ {}_3F_2(-k, b, a+b-\alpha-\beta; a+b-\beta, a+b-\alpha; 1)\,, \tag{B.8}$$

where now the hypergeometric function is reduced to a finite sum of products of Γ functions.

Thus we find the following residues $R'(2\alpha')$ of $\Gamma(2\alpha, 2\beta; 2\alpha', 2\beta')$, $G(2\alpha, 2\beta; 2\alpha', 2\beta'; b)$ for $2\alpha' = -2l$ and $2\alpha' = -(2l+1)$ respectively:

$$R'(2l) = \frac{(-1)^l}{l!}\, 2^{-2l-4+2(\beta'+\alpha+\beta)}\pi^{-1/2}$$

$$\times\, \Gamma\begin{pmatrix} \beta', \beta'+\frac{1}{2}-l,\ \alpha+\beta+\frac{1}{2},\ \alpha,\ \beta,\ b, \frac{1}{2}+b-\alpha-\beta \\ \frac{1}{2}+b-\alpha,\ \frac{1}{2}+b-\beta \end{pmatrix}$$

$$\times \sum_{k=0}^{l} \frac{(-l)_k(\beta)_k}{(\frac{1}{2})_k k!}\; {}_3F_2(-k, b, \tfrac{1}{2}+b-\alpha-\beta; \tfrac{1}{2}+b-\alpha, \tfrac{1}{2}+b-\beta; 1)\,, \tag{B.9}$$

$$R'(2l+1) = \frac{(-1)^l}{l!}\, 2^{-2l-4+2(\beta'+\alpha+\beta)}\pi^{-1/2}$$

$$\times\, \Gamma\begin{pmatrix} \beta'+\frac{1}{2},\ \beta'-l,\ \alpha+\beta+\frac{1}{2},\ \alpha+\frac{1}{2},\ \beta+\frac{1}{2},\ b,\ \frac{1}{2}+b-\alpha-1 \\ 1+b-\alpha,\ 1+b-\beta \end{pmatrix}$$

$$\times \sum_{k=0}^{l} \frac{(-l)_k(\beta'+\frac{1}{2})_k}{(\frac{3}{2})_k k!}\; {}_3F_2(-k, b, \tfrac{1}{2}+b-\alpha-\beta; 1+b-\alpha, 1+b-\beta; 1), \tag{B.10}$$

with

$$(x)_k = \frac{\Gamma(x+k)}{\Gamma(x)}\,.$$

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